EXAMINERS REPORT

MATHEMATICS - II HIGHER SECONDARY SCHOOL CERTIFICATE

ANNUAL EXAMINATION 2018



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FEDERAL BOARD OF INTERMEDIATE &
SECONDARY EDUCATION, ISLAMABAD
(PAKISTAN)

PART I: STATISTICAL INFORMATION

The examination comprising a paper of 100 marks was administered to the students who had completed their two years of academic education at Higher Secondary School Certificate (HSSC) level. Question paper was divided into three sections each containing different types of questions, namely: "Section A" consisting of question number one, having twenty compulsory structured part questions - Multiple Choice Questions (MCQs) of one mark each (20% weighting); "Section B comprising fourteen open ended questions with limited cognitive demand - Short Response Questions (SRQs) taken from the prescribed book with 40 percent weighting and candidates were required to answer any ten questions out of them of their own choice carrying four marks each; "Section C" consisting of seven open ended questions with greater cognitive demand - Extended Response Questions (ERQs) out of which students were required to answer any five questions of eight marks each with proportionate weighting of 40 percent of the paper. Time duration of the paper was three hours.

A total of 24647 candidates appeared in this paper during the annual examination 2018 and out of them 14961 (60.7 percent) passed the examination with the grade percentage distribution as summarized in Table 1:

Table 1: Grade-wise distribution of candidates

Grade	Students	Percentage of Grade
A1	961	3.9
A	1799	7.3
В	2489	10.1
С	3156	12.8
D	3894	15.8
Е	2662	10.8
F	8947	36.3
Absent	739	3.0
Total	24647	100

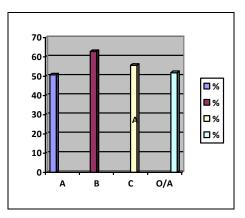
PART II: GENERAL COMMENTS

The performance of the candidates in handling "Section A" was the weakest in terms of attainment of relatively the lowest marks in this section when compared with other sections. Generally the questions in this section comprised of mixed cognitive levels in that there were 4(20%) marks questions of knowledge level, which required mere recall of equations, terms and procedures, 11(55%) marks questions of understanding level, which required some understanding of concepts, and 5(25%) marks questions were of application level. The students performed better in knowledge level MCQs while they faced some difficulty in answering the questions which required understanding of concepts, particularly their application. However, their performance was good where the correct answers were either obvious or distracters were weaker. The performance of the candidates was the lowest in part questions (vii), (ix), (x) and (xii) of understanding level and part questions (ii), (iv) and (v) of application level (Question Paper Version 1871) where a greater percentage selected at least one distracter in preference to the correct answer. This suggests that either the requirements of questions were not properly understood by majority of the candidates or they were not well prepared for the examination. The candidates secured on the average 7.77 (38.84%) marks in this section. The performance of the students in answering the part questions of "Section B", which consisted of five questions 20 (37.41%) marks of knowledge, nine questions 32 (57.14%) marks of understanding and only one part question 4 (7.15%) marks of application levels, was the best when compared with other sections of the paper. They generally faced difficulty in fully answering the questions which involved understanding of concepts, despite the fact that all questions were taken word by word from the book, which are usually discussed in the classroom by the teachers. The candidates secured on the average 20.83 (52.07%) marks in this section. The part questions (vi) and (xii) of understanding level, (ix) of application level and (xiii) of knowledge level were the least choice questions attempted by less than 40 percent of the candidates and their performance in these questions was the lowest. This suggests that the candidates had generally carried out selective study for the examination. The "Section C" comprised of seven extended response questions (ERQs) carrying 8 marks each of mixed cognitive levels with an overall bias towards higher cognitive levels. There were around 8 (14.29%) marks portions of knowledge, 16 (28.57%) marks portions of understanding and 32 (57.14%) marks portions of application levels. The general performance of candidates was lower in this section as compared to "Section B" mainly due to higher cognitive demands and they on the average secured 17.54 (43.84%) marks. The preference of candidates varied in the selection of questions and out of these seven questions, the lowest choice three questions attempted were Question 5 by 32.2%, Question 7 by 6.7% and Question 8 by 9.2% whereas the three popular choice questions were Question 3 by 53.7%, Question 7 by 77.1% and Question 9 by 47.2% from this section. This suggests that generally the candidates did not prepare well the chapter 4 (Introduction to Analytical Geometry) and chapter 5 (Linear Inequalities) whereas they had prepared well chapter 3 (Integration) and chapter 7 (Vectors). The overall performance of candidates was in the middle of the three sections having performance index as 55.65% and had secured on the average 17.54 (43.84%) marks. Almost 60.7% of the candidates passed the examination mainly due to the questions of lower level cognitive demands besides this all questions were given word by word from the book, which the students had practiced before in the classrooms and these were mere recall (knowledge level) questions for them. The section-wise performance of candidates is as indicated in Table 2 and Fig 1.

Table 2: Accumulative performance.

Table 2. Recalliable performance.									
Section	Performance Index	%							
А	0.51	50.73							
В	0.63	62.9							
С	0.56	55.65							
O/A	0.52	51.84							

Fig. 1 Accumulative performance in all sections.



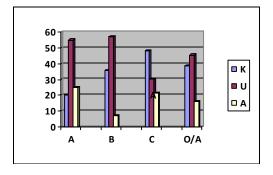
In certain questions where understanding of concepts or their application was required for answering, the candidates faced some difficulty in comprehending the complete requirements especially in the case of open ended questions contained in sections "B and C". In addition, the part questions (MCQs) in "Section A" included questions which required more than 25 minutes, given in the question paper, for properly answering as these contained 80 percent of the questions of understanding and application levels which presumably resulted in the lowest performance of the candidates. The overall performance of candidates varied from section to section, the best in case of "Section B" (SRQs), followed by "Section C" (ERQs) and

comparatively the lowest in "Section A" (MCQs). The section-wise marks allocated to different question items according to their cognitive levels are given in Table 3 and fig 2.

Table 3: Marks % - Cognitive Level

	•		
Section	Knowledge (K)	Understanding (U)	Application (A)
Α	20	55	25
В	35.71	57.14	7.15
С	48.21	30.36	21.43
O/A	38.64	45.45	15.91

Fig.2 Marks % allocated in all sections.



Areas Noted for Improvement of Various Stakeholders

Analysis of question paper, answer scripts, performance and observation/comments of Head and Sub-examiners led to the identification of following deficiencies in the examination system:

Question Paper

- i) Question paper setter used appropriate command words while writing the question items for different sections of this paper. The question items were biased towards higher cognitive levels due to which candidates faced difficulty in answering. The performance of the candidates is just satisfactory presumably due to time constraints and selective study by the candidates for the examination.
- ii) Inability of paper setter to construct questions where students were required to be tested for understanding of concepts and their application in a novel situation to discriminate those who understand the concepts and can apply them in a given situation from those who can reproduce through memory only. The questions given in the paper were reproduced verbatim from the book, due to which the candidates were tested for the preparation for the examination only. The candidates, where teachers had thoroughly covered the questions given in the book, performed better when compared with those where it was partially covered and left to the students to practice at their own.
- iii) Inability of paper setter to keep in view while constructing MCQs "Section A" the time constraints for the candidates for answering especially the part questions 2), 4), 5), 7), 9), 10) and 12). It is presumed that majority of the candidates answered these questions by

guessing the correct answer due to which they picked up the distracters in preference to the correct answers.

Student Learning Objectives and Examination Grid

The question paper was prepared from the contents of the text book only and the Student Learning Objectives (SLOs) of the course were ignored; even the MCQs were reproduced. To facilitate the paper setters and the teachers in the institutions, the board had issued the SLOs of the subject and a model question paper which was to be followed in its true letter and spirit including the implied examination grid. The question paper was prepared, following the pattern of the model question paper only, without ensuring the required percentage of question items of different difficulty levels (examination grid). There is a requirement to issue the Examination Grid for this paper to all stakeholders to ensure standardization and compliance by all concerned.

Marking Key

v) Marking Key which was issued to the sub-examiners to ensure uniformity in marking of answer scripts by them was appropriately worded and prepared with minor details of award of marks. It is felt that marking of this paper was by and large reliable. The Marking Key is the backbone of the entire evaluation system and needed to be elaborate explaining the requirements along with award of marks for each completely or partially answered portions of a question. The preparation of Marking Key is the responsibility of the Question Paper Setter to ensure the uniformity of marking at all marking centres but here in the instant case it was prepared by the Head Examiner of the marking team at Islamabad centre.

Markers

- vi) The inability of markers to evaluate accurately the answer books is evident from the non-uniformity in award of marks in that, similar answers of different questions were awarded differently, certain incorrect steps were ignored and some incomplete answers were given full credit. This suggests incompetence of markers to comprehend fully the requirements of question items and they also ignored the details given in the marking key and instead generally depended upon their own experience and understanding in award of marks. In addition, the disconnect between the question paper setter and the heads of the marking teams who prepared the marking key keeping in view the requirements as ascertained by them contributed to the non-uniformity in marking.
- vii) The purpose of examination is twofold; one is to determine the students who have successfully acquired the needed knowledge and skills required at their level and can apply certain concepts in a novel situation and the second one is to place the successful candidates

in different grades according to their attainment levels. The question paper, requiring to recall solutions of the questions as given in the text book, ignoring the time constraints and a disconnect between the question paper setter and the head examiner in preparation of marking key besides unreliable marking of answer books by inexperienced markers negate the very purpose of examination and it appears mere an exercise. The examination in its present form will promote rote learning by the students only without testing their comprehension and application skills.

Recommendations

- viii) There is a requirement to change existing system of paper setting and marking procedure of answer books at the board level to ensure validity and reliability of the examination. Parameters (question paper pattern and examination grid) be laid down as per international standards of the subject and must be followed for at least five years to ascertain and compare accurately the progress made by the students in teaching and learning in the institutions, against a uniform assessment standard. In the present situation, in the absence of a reliable examination system progress shown through the marks and grades attained by the students remains questionable.
- ix) Efforts should be made at the board level to improve quality of question papers in terms of their face, contents, construct and criterion validity through capacity building of test item writers and paper setters.
- x) Workshops may be arranged and refresher courses be run for the sub-examiners besides inducting more competent markers available into the system using IT technology.
- xi) Efforts of FBISE towards establishment and strengthening of question bank, prepared by the professionals besides continuous additions into it may help to reduce issues of composition and construct of quality question papers. As a start point question papers set by internationally known credible boards during their previous years examinations at HSSC/equivalent levels may be reviewed by a committee of experts of each subject to lay down the guiding principles for preparing the test items.

PART III: QUESTION SPECIFIC COMMENTS

SECTION - A

Question Number One, an exclusive question in this section consisting of 20 compulsory Multiple Choice Questions "(MCQs)" as part or sub-questions was attempted by all (99.80 %) examinees present in the examination halls and the overall achievement of examinees in this section is summarized in Table 4 below:

Table 4: Distribution of candidates against different levels of achievement

Marks	1-4	5-8	9 – 12	13 – 16	17 – 20	Mean (Percentage)
Percentage	8.25	33.28	40.21	12.74	5.52	7.77 (38.84)

Mean marks obtained in this section by the candidates are 7.77 (38.84 %) with an overall achievement of around 50.73 percent which is the lowest when compared with their overall performance in sections "B and C". Overall achievement of 50.73 percent is biased towards lower side. Around 41.53 percent of candidates are positioned in the two lowest quintiles of marks (less than 40 %) while 5.52 percent are in the highest quintile of marks (above 80 %) and around (40.21 %) are positioned in the third quintile of marks i.e. between 40 and 60 % in this section.

Decomposition analysis of part questions revealed that except for four part questions (3, 8, 15 and 20) which were of "Knowledge Level" where students were required to recall some formulae, terms and expressions whereas the remaining sixteen part questions were of understanding of concepts and their applications. Those students who were good in memorizing or recalling required information from the contents of the book performed relatively better. Since all questions were compulsory, without negative marking for incorrect answers, so everyone attempted all part questions. Question and response analysis of this section is summarized in Table 5 below:

Table 5: Question and response analysis with option chosen against each question (Version Number 1871)

					Com	ments	{cognitiv	e level	(CL),	
Question	% A	% B % C % D			discrimination index (DI), poor distracter (PD), strong distracter (SD), facility index					
					(FI) CL	DI	FI	PD	SD	
1	44.24	12.13	20.51	14.02	U	DI _(0.27)	0.44		C	
	44.34		29.51	14.02						
2	20.22	23.75	30.13	26.04	Α	0.24	0.20		A,B,C	
3	22.62	10.38	49.57	17.56	K	0.32	0.49		A,D	
4	26.39	31.75	19.59	22.68	Α	0.36	0.26		B,C,D	
5	13.33	20.55	49.03	15.89	Α	0.10	0.20		C,D	
6	30.56	31.74	27.06	10.88	Α	0.48	0.31		A,C	
7	34.21	19.19	29.75	15.41	U	0.32	0.29		A,B,D	
8	23.52	51.41	10.84	14.07	K	0.53	0.51		A	
9	16.53	36.05	11.60	34.75	U	0.42	0.34		A,C	
10	15.87	29.17	34.34	18.75	U	0.31	0.29		A,C,D	
11	13.72	25.54	25.53	32.49	U	0.44	0.32		В,С	
12	39.96	39.60	11.86	7.62	U	0.55	0.39	D	A	
13	10.36	19.17	17.32	50.30	U	0.51	0.50		В,С	
14	10.41	65.61	16.22	7.12	K	0.52	0.65	D	C	
15	37.68	20.63	30.48	8.74	U	0.54	0.37	D	В,С	
16	35.54	26.85	18.21	16.86	U	0.39	0.35		B,C,D	
17	29.48	27.42	21.45	18.52	Α	0.32	0.29		B,C,D	
18	17.44	39.33	24.42	15.78	U	0.43	0.39		A,C,D	
19	13.57	17.66	56.38	10.45	U	0.58	0.56		В	
20	61.98	4.23	15.69	17.12	K	0.35	0.61	В	C,D	

^{*}The correct answer is indicated by shading

Cognitive level: Knowledge (K), Understanding (U), Application (A)

Facility Index (FI): It ranged between 20 and 65 percent in all part questions. The facility indices in six part questions (30 %) is between 20 and 30 percent, seven part questions (35 %) is between 31 and 40 %, five part questions (25 %) is between 41 and 60 % and in only two part questions, 14 and 20, it is 65 and 61 percent respectively. The overall FI is 58.73 percent whereas 53.97 percent of the candidates are in the lower two quintiles (less than 40 percent marks) and only 13.32 percent candidates are in the upper two quintiles (above 60 percent marks) for this question.

Discrimination Index (DI): In order to correlate the performance of the candidates in a subquestion with their overall score (combined 20 sub-questions) discrimination index is

calculated by taking 27 percent examinees each from upper and bottom sub-groups of this cohort, keeping in view the sample constraints, four different versions of question papers, in that version 1871 has been taken for analysis as it was attempted by maximum number of candidates 9238 (37.64% of total candidates). The positive value of DI, for all questions, indicates that the requirements of the questions were well understood by the examinees. On the basis of DI values, **eleven** items are found very good test items for having values equal or more than 0.4, **seven** items are reasonably good with DI value ranged from 0.30 to 0.39, **one** is marginal item with DI value ranged from 0.2 to 0.29 and **one** is the weakest (Sub-question 5 with facility index 20 %) with DI value 0.10, (Ebel and Frisbie, 1986). Amongst these, **fourteen** MCQs are found ideal questions having difficulty (facility) index range between 0.3 and 0.7 with DI value greater than 0.24.

Discrimination coefficient measuring effectiveness of each distracter has also been determined using similar formula as suggested by Nitko and Hsu (1984). Accordingly, seven part questions (2, 4, 7, 10, 16, 17 & 18) 35 percent of MCQs were found having three effective distracters, suggesting that either the questions were not understood clearly by the candidates or they were not well prepared, eight part questions (3, 5, 6, 9, 11, 13, 15 & 20) 40 percent of MCQs were having two effective distracters whereas five part questions (1, 8, 12, 14 & 19) 25 percent MCQs were having only one good distracter.

SECTION - B

Question number two was the single question in this section consisting of 14 Short Response Questions (SRQs) as sub-questions of 4 marks each and the candidates were required to answer any 10 out of them. Most of the candidates attempted four of knowledge level questions (i), (ii), (v) and (xiv), randomly attempted five out of the eight understanding level questions (iii), (iv), (vii), (viii), (viii), (x), (xi) and (xii). A small percentage around 20 percent attempted application level part question (ix), in which candidates were required to find the angle between two lines when their slopes were given. The candidates generally avoided the easier questions from the Conic Section and selected those questions from this section which were having greater cognitive demand. The facility index (FI) of question 2 is 0.75 and its discrimination index (DI) is 0.95. The overall performance of the candidates in this section was the best where they on the average secured 20.83 (52.07 %) marks and 1.14 % out of them secured 100 % marks

Overall achievement of examinees in this section is summarized in Table 6 below:

Table 6: Distribution of candidates against different levels of achievement

Marks	1-8	9 – 16	17 – 24	25 – 32	33 – 40	Mean (Percentage)
Percentage	9.45	22.01	29.80	21.60	17.13	20.83 (52.07)

Decomposition analysis of sub-questions revealed that 35.71 percent questions in this section were of knowledge, 57.14 percent of understanding and 7.14 percent of application levels. Those candidates who were good in conceptual understanding of the subject performed relatively better. Since candidates had to choose any ten sub-questions out of fourteen so there was a general trend to select questions requiring lower cognitive demand. Further, the least choice questions were (ix), (xii) and (xiii) from the chapters on "Introduction to Analytic Geometry" and "Conic Section" irrespective of their cognitive levels which suggests that the candidates had not prepared these chapters for the examination. However, part question (vi) of understanding level was the least attempted by the candidates presumably they faced difficulty in interpreting |x|, as they had done better in other questions from the chapter on "Integration". Question and response analysis of this section is summarized (most popular "K", on choice "U" and the least attempted "A") in Table 7 below:

Table 7: Question and response analysis against each sub-question

Sub-question	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)	(xii)	(xiii)	(xiv)
Cognitive	K	K	U	U	K	U	U	U	A	U	U	U	K	K
Level (CL)														
Attempted	75-80	50-55	90-95	90-95	75-80	25-30	70-75	90-95	30-35	80-85	70-75	30-35	35-40	85-90
(%)														

Specific Responses to parts of the Question 2:

Question Part (i)

If
$$f(x) = \begin{cases} x+2, & x \le -1 \\ c+2, & x > -1 \end{cases}$$
 find "c" so that $\lim_{x \to -1} f(x)$ exists

Question asked required the candidates to explain that,

If
$$\lim_{x \to 1} f(x)$$
 exists then
$$\lim_{x \to -1} f(x) = \lim_{x \to -1} f(x)$$

Candidates were asked to calculated the value of "c" by substituting limiting value of x=-1 and equating the function on both sides at the boundary

$$\lim_{x \to -1} (x+2) = \lim_{x \to -1} (c+2)$$

$$-1+2=c+2$$

$$c=-1$$

Question asked was of knowledge level requiring the candidates to recall the condition for continuity of a function when its limits exist at a point. Almost 75 - 80 percent of the candidates attempted this question. However, some of them were confused and faced difficulty in finding value of "c". On the average they performed extremely well in this question and secured around 3-3.5 marks.

Question Part (ii)

Find the value of $\frac{dy}{dx}$ using the first principle if $y = \frac{1}{\sqrt{x+a}}$

Candidates were required to derive an expression for the increment in the function y with the corresponding increment in x.

$$y = \frac{1}{\sqrt{x+a}} \implies y + \delta y = \frac{1}{\sqrt{x+\delta x+a}} \implies \delta y = \frac{\sqrt{x+a} - \sqrt{x+\delta x+a}}{\sqrt{x+\delta x+a}\sqrt{x+a}}$$

They were expected to simplify the equation for increment in the function y with respect to x as under,

$$\delta y = \frac{(x+a) - (x+\delta x + a)}{\sqrt{x+\delta x + a}\sqrt{x+a}\left(\sqrt{x+a} + \sqrt{x+\delta x + a}\right)}$$

$$\frac{\delta y}{\delta x} = \frac{-1}{\sqrt{x + \delta x + a}\sqrt{x + a}\left(\sqrt{x + a} + \sqrt{x + \delta x + a}\right)}$$

Candidates were asked to express the relation by taking limit as δx approached zero.

$$\frac{dy}{dx} = \frac{-1}{\sqrt{x+a}\sqrt{x+a}\left(\sqrt{x+a} + \sqrt{x+a}\right)} = -\frac{1}{2}(x+a)^{-\frac{3}{2}}$$

Question asked was of knowledge level requiring the candidates to differentiate the given function using first principle. They were required to recall the procedure for differentiation. Almost 50-55 percent of the candidates attempted this question and majority of them successfully differentiated the function except for a few 10-15 percent candidates who faced difficulty in taking the limit as δx approaches zero. On the average they performed very well and secured 2.5–3 marks in this question.

Question Part (iii)

If
$$y = tan(p tan^{-1}x)$$

then show that,
 $(1 + x^2) y_1 - p (1 + y^2) = 0$

Candidates were expected to differentiate the function w.r.t. x to get the following equation.

$$y_1 = \sec^2(p \, \text{Tan}^{-1} x) \cdot \frac{p}{1 + x^2}$$

They were required to use trigonometric identity and rearrange the equation as under,

$$(1+x^{2})y_{1} = p[1+\tan^{2}(p \operatorname{Tan}^{-1}x)]$$

$$(1+x^{2})y_{1} = p(1+y^{2}) \implies (1+x^{2})y_{1} - p(1+y^{2}) = 0$$

Question asked was of understanding-cum-application level requiring the candidates to first differentiate the given function w.r.t. x which involved trigonometric function and then using the trigonometric identities they were required to simplify and rearrange the equation as required. Almost 90-95 percent of the candidates attempted this question and majority out of them successfully solved the question except for a few (10–15%) candidates who faced difficulty in differentiating the trigonometric function correctly. The candidates performed extremely well in this question and on the average they secured 3.5-4 marks.

Question Part (iv)

Find the intervals in which the function

$$f(x) = x^2 + 3x + 2$$
; $x \in (-4, 1)$ is

- a. Increasing
- b. Decreasing

Question required the candidates to differentiate the function w.r.t. x and apply conditions for increasing and decreasing functions as follows,

$$f(x) = x^2 + 3x + 2$$
; $x \in (-4, 1)$
 $f'(x) = 2x + 3$
 $\Rightarrow 2x + 3 > 0 \Rightarrow x > -\frac{3}{2}$

They were also required to state that the function f(x) is increasing when its slope or derivative f'(x) > 0 which implies,

i.e.
$$f(x)$$
 is increasing in the interval $\left\{-\frac{3}{2}, 1\right\}$

They were also required to state that the function f(x) is decreasing when its slope or derivative f'(x) < 0 which implies,

$$\Rightarrow 2x + 3 < 0 \Rightarrow x < -\frac{3}{2}$$

i.e. function
$$f(x)$$
 is decreasing in the interval $\left\{-4, -\frac{3}{2}\right\}$

Question asked was of **understanding level** in which the candidates were required to find the intervals in which the given function increases and decreases. Almost 45-50 percent of the candidates attempted this question and majority (80-85%) out of them successfully answered it correctly. However, some (15-20%) of the candidates did not co-relate the slope of the function with increasing or decreasing function. The overall performance of the candidates who attempted this question was quite good who secured on the average 3–3.5 marks.

Question Part (v)

Evaluate
$$\int \sqrt{1-\cos 2x} \ dx$$
 (where 1- $\cos 2x > 0$)

The question asked required the candidates to use the trigonometric identity for $\cos 2x$

$$\int \sqrt{1-\cos 2x} \ dx = \int \sqrt{1-\cos^2 x + \sin^2 x} \ dx$$

They were expected to simplify by using the trigonometric identity to obtain the relation

$$=\int \sqrt{2\sin^2 x} \ dx$$

Candidates were required to integrate by using formula

$$= -\sqrt{2}\cos x + c$$

Question asked was of knowledge level which required the candidates to recall the trigonometric identity to simplify the integrant before integration. Besides this other methods were also available. Almost 75-80 percent candidates attempted this question and out of them around 40-45 percent solved the question by applying alternate methods including direct formula for integration. They secured on the average 3–3.5 marks.

Question Part (vi)

Evaluate
$$\int_{-1}^{2} (x + |x|) dx$$

Candidates were expected to break the limits of integration as under,

$$= \int_{-1}^{0} (x + |x|) dx + \int_{0}^{2} (x + |x|) dx$$

They were also expected to substitute the values of |x| keeping the limits of integration in view,

$$= \int_{-1}^{0} (x-x)dx + \int_{0}^{2} (x+x)dx$$

Candidates were required to integrate by using the formula and substitute the limits to evaluate the required value as under,

$$= \int_{-1}^{0} (0) dx + \int_{0}^{2} (2x) dx = \left[x^{2}\right]_{0}^{2}$$

$$=2^2-0^2=4$$

Question asked was of **knowledge-cum-understanding level** aimed to test the ability of the candidates to substitute the values of |x| keeping the limits of integration in view and were also required to integrate the given function by recalling the formula and procedure. This question was one of the least choice questions and was attempted by around 25-30 percent of the candidates who secured on the average 1-1.5 marks.

Question Part (vii)

Solve the following differential equation xdy + y(x-1) dx = 0

Question required the candidates to rearrange the equation by separating the variables and to integrate both sides by using the formulae as under,

$$xdy + y(x-1)dx = 0 \Rightarrow \int \frac{1}{y} dy = \int \frac{1-x}{x} dx$$

$$\Rightarrow \int \frac{1}{y} dy = \int \left(\frac{1}{x} - 1\right) dx \Rightarrow \ln y = \ln x - x + \ln c$$

Candidates were also required to simplify the equation by using logarithmic identities as under,

$$\Rightarrow \ln y = \ln cx - x \Rightarrow \ln \frac{y}{cx} = -x$$

Candidates were expected to take antilog of the terms on both sides of the equation,

$$\Rightarrow \frac{y}{cx} = e^{-x} \Rightarrow y = cxe^{-x}$$

Question asked was of **knowledge-cum-understanding level** aimed to test ability of the candidates to rearrange the equation by separating the variables and to integrate by recalling the formula. They were also expected to take antilog of the resultant function and to simplify it by rearranging and applying the logarithmic identities. This question was attempted by around 70-75 percent of the candidates and majority of them answered correctly up to integration level. However, some (20-25) percent out of them faced difficulty in applying logarithmic identities. The overall performance of the candidates was quite good who on the average secured 3-3.5 marks.

Question Part (viii)

The xy-coordinates are rotated about the origin through an angle of measure $\theta=30^{\circ}$ and the new axes are OX and OY. Find the xy-coordinates of point P with the given XY- Coordinates of point P as (-5,3)

Question required the candidates to give the equations of rotation of axes as under,

$$X = x \cos \theta + y \sin \theta$$
 & $Y = y \cos \theta - x \sin \theta$

They were also required to substitute the value of the angle θ =30° and express the values of y in terms of x

$$y = -10 - \sqrt{3}x$$
 & $y = \frac{6+x}{\sqrt{3}}$

Candidates were expected to argue that the right hand sides of both equations must be equal so by equating found the value of x coordinate.

$$-10 - \sqrt{3}x = \frac{6+x}{\sqrt{3}}$$
 \Rightarrow $x = \frac{-3 - 5\sqrt{3}}{2}$

Candidates were required to substitute the value of x coordinate in any of the two equations to find the value of y coordinate and hence the coordinates of the point P.

$$y = \frac{3\sqrt{3} - 5}{2} \qquad P\left(\frac{-3 - 5\sqrt{3}}{2}, \frac{3\sqrt{3} - 5}{2}\right)$$

Question asked was of **understanding level** in which candidates were required to find the coordinates of a point, when the xy-coordinates are rotated through an angle of 30° about the origin. This question was one of the most popular choice questions which was attempted by almost 90-95 percent of the candidates who on average secured 3-3.5 marks.

Question Part (ix)

Find the angle from the line with slope $\frac{-7}{3}$ to the line with slope $\frac{5}{2}$.

Question required the candidates to assume the slopes of two lines as $l_1 = m_1$ and $l_2 = m_2$ and to give the equation of the angle between them as

$$\tan\theta = \frac{m_2 - m_1}{1 + m_2 m_1}$$

Candidates were also required to substitute the values of m_1 and m_2 to calculate the value of $\tan \theta = -1$ and to find the value of the angle from the table as $\theta = 135^{\circ}$

Question asked was of **Application level** aimed to test ability of the candidates to find the angle between the lines when their slopes are given. This was one of the least popular questions attempted by 30-35 percent of the candidates. However, 70-75 percent out of them successfully answered the question. They, on the average secured 2.5-3 marks in this question.

Question Part (x)

Graph the feasible region of system of linear inequalities;

$$3x + 2y \ge 6$$
; $x + y \le 4$; $x > 0$, $y > 0$

Also find the corner points.

Question asked required the candidates to draw the graphs for the corresponding equations and to find their intercepts with the coordinate axes. The corresponding equations are,

$$3x + 2y = 6$$
 and $x + y = 4$

And their intercepts are: (0,3), (2,0), (0,4), (4,0)

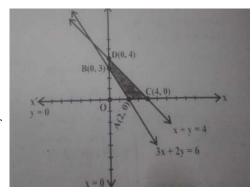
Candidates were expected to take origin (0,0) as the test point to determine the region by substituting coordinates of the test point in the equations

$$3(0) + 2(0) > 6$$
, which is false,

They were expected to argue that the Solution Region lies opposite to the test point 0 + 0 < 4, which is true

Further, the candidates were expected to explain that the Solution Region lies towards the test point and were also required to give the corner points as (0,3), (2,0), (0,4), (4,0). They were also required to graph the region.

Question asked was of **understanding level** aimed to test ability of the candidates to draw the graph of the equations and to determine the feasible region of the given inequalities. This question was one of the most popular choice questions attempted by almost 80–85



percent of the candidates who answered it very well except for some (15-20%) who faced difficulty in finding the corner points. They on the average secured 2.5-3 marks.

Question Part (xi)

Find the centre and radius of the circle

$$x^2 + y^2 + 14x - 10y = 0$$

This question required the candidates to rearrange equation in the form of squares of x and y

$$(x^2+12x+36)+(y^2-10y+25)=36+25$$

$$(x+6)^2 + (y-5)^2 = (\sqrt{61})^2$$

They were required to interpret the coordinates of the centre and radius of the circle from the equation as: centre:(-6,5). $radius:\sqrt{61}$

Question asked was of **understanding level** which required the candidates to rearrange the given equation in the form of standard equation for a circle to determine the coordinates of the centre of the circle and its radius. This question was attempted by almost 70–75 percent of the candidates and their performance was extremely well who on the average secured 3–3.5 marks.

Question Part (xii)

Write equation of tangent to parabola, $x^2 = 16y$, at the point whose abscissa is 8.

This question required the candidates to calculate the corresponding y coordinate from the equation for the coordinate x = 8 and hence the tangent point (8,4)

They were expected to differentiate the equation of parabola w.r.t. x to find the slope, parabola: $x^2 = 16y$

Differentiating w.r.t.x
$$\Rightarrow \frac{dy}{dx} = \frac{x}{8}$$

Candidates were required to calculate the slope of the tangent at the point (8.4)

then slope of
$$t = \left(\frac{dy}{dx}\right)_{(8,4)} = \frac{8}{8} = 1$$

They were required to give the equation of the tangent as asked,

$$y-4=1(x-8)$$
 $\Rightarrow x-y-4=0$

Question asked was of **understanding level** in which candidates were required to write the equation of tangent to the given parabola at the point of given coordinates. This was one of the least attempted questions. Around 30-35 percent of the candidates attempted this question and majority (70-75%) of them faced difficulty in finding the slope of the tangent. Overall their performance was poor who on the average secured 1-1.5 marks.

Question Part (xiii)

Show that 10xy + 8x - 15y - 12 = 0; represent a pair of straight lines

In this question the candidates were required to rearrange and carry out factorization of the equation as under,

$$10xy - 15y + 8x - 12 = 0$$

$$(2x-3)(5y+4)=0$$

Candidates were required to argue that the solution set of pair of equations is as under, 2x-3=0; 5y+4=0

Question asked was of **knowledge level** in which the candidates were required to recall the procedure of factorization. This was one of the least attempted questions and around 35–40 percent of the candidates attempted this question who on the average secured 2–2.5 marks.

Question Part (xiv)

If
$$\underline{\mathbf{u}} = 2\underline{\mathbf{i}} - \underline{\mathbf{j}} - \underline{\mathbf{k}}$$
 and $\underline{\mathbf{v}} = 4\underline{\mathbf{i}} + 2\underline{\mathbf{j}} - \underline{\mathbf{k}}$, then find the vector $\underline{\mathbf{u}} \times \underline{\mathbf{v}}$.

This question required the candidates to carry out vector product of two vectors by determinant method

$$\underline{\mathbf{u}} = 2\underline{\mathbf{i}} - \underline{\mathbf{j}} - \underline{\mathbf{k}}, \quad \underline{\mathbf{v}} = 4\underline{\mathbf{i}} + 2\underline{\mathbf{j}} - \underline{\mathbf{k}}$$

$$\underline{u} \times \underline{v} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & -1 & -1 \\ 4 & 2 & -1 \end{vmatrix}$$

They were expected to express the resultant vector by opening the determinant and simplifying as under,

$$\underline{u} \times \underline{v} = 3\underline{i} - 2\underline{j} + 8\underline{k}$$

Question asked was of **knowledge level** in which the candidates were required to recall the procedure for finding the vector product of two vectors. Almost 85–90 percent candidates attempted this question and secured on the average 3–3.5 marks.

SECTION - C

This section was comprised of seven "Extended Response Questions (ERQs)" with equal (8) marks each and candidates were required to attempt any five out of the given seven questions. Each question consisted of mixed cognitive levels, in that 8 (14.29%) marks portions were of knowledge, 16 (28.57 %) marks portions of understanding and 32 (57.14%) marks portions were of application levels. The overall achievement of the candidates in this section was in the middle of the three sections and they on the average secured 17.54 (43.84%) marks, mainly due to the bias of questions towards higher cognitive levels (understanding and application levels). Though all the questions were taken verbatim from the text book, which are usually covered in classroom teaching, yet performance of examinees varied from question to question according to their cognitive levels. Out of a total of 91.55 percent examinees who attempted this section only around 18.69 percent succeeded in attempting five required questions from this section. Out of them around 19.59 percent attempted four questions, 20.27 percent attempted three questions, 23.20 percent attempted two questions and 18.24 attempted only one question. Approximately around 2.48 percent of the candidates secured 100 percent marks in this section. Overall achievement of examinees in this section is summarized in Table 8 below:

 Table 8 :
 Distribution of candidates against different levels of achievement

Marks	1-8	9 – 16	17 – 24	25 – 32	32 – 40	Mean (Percentage)
Percentage	17.91	29.17	21.62	19.37	11.94	17.54 (43.84)

Decomposition analysis of portions of questions revealed that a total of 8 (14.29%) marks portions were of knowledge, 16 (28.57%) marks portions of understanding and 32 (57.14%) marks portions of application levels. Those students who were good in conceptual understanding of the subject and had carried out thorough preparation for the examination performed relatively better. Since the candidates had to choose any five out of the seven given questions so there was a general trend to select questions requiring lower cognitive demand. In addition majority of them avoided questions taken from the chapters on "Introduction to Analytic Geometry" and "Conics Section" irrespective of their cognitive level which reflects a general trend of leaving these chapters on choice. Question and response analysis of this section is summarized in the Table 9 below:

Table 9: Question-wise Marks & % - Section C

Question	Cognition	Average Marks	Facility Index	Discrimination	Attempted
	Level	(%)	(FI)	Index (DI)	(%)
	K = 25 %	6.49			
3	U = 50 %	(81.09 %)	0.91	0.34	53.71
	A = 25 %				
	K = 75 %	5.13			
4	U = 25 %	(64.07 %)	0.77	0.61	45.98
	K = 25 %	4.45			
5	U = 37.5 %	(55.57 %)	0.82	0.35	32.16
	A = 37.5 %				
	K = 62.5 %	6.60			
6	U = 12.5 %	(82.45 %)	0.94	0.15	77.11
	A = 25 %				
	K = 25 %	5.80			
7	U = 37.5 %	(72.54 %)	0.80	0.44	6.29
	A = 37.5%				
	K = 75 %	4.78			
8	U = 25 %	(59.71 %)	0.70	0.60	9.69
	A = - %				
	K = 50 %	6.08			
9	U = 25 %	(75.96 %)	0.82	0.51	47.22
	A = 25 %				

Facility Index (FI): It ranged from 0.70 to 0.94 in all questions. Facility index of question number 8, involving the centre, foci, eccentricity, vertices and equations of directrices of the given hyperbola was the lowest 0.7. FI above 0.70 suggests that the candidates who attempted any question out of the given seven questions performed extremely well in all of them from this section.

Discrimination Index (DI): In order to correlate the performance of the candidates in questions (3-9) of "Section C" with their overall score (in questions 2-9) discrimination index has been calculated by taking 27 percent of the examinees each from upper and bottom subgroups of this cohort, keeping in view the sample constraints, results of question 1 ("Section A") and questions (2-9) "Sections B and C" received from two different sources and the overall higher FI. The positive value of DI, for all questions, indicates that the requirements of the questions were well understood by the examinees. On the basis of DI values six questions are found very good test items for having values equal or more than 0.4. The DI values of questions 3 and 5 are 0.34 and 0.35 respectively are fairly good test items whereas DI value of question 6 is only 0.15 and its corresponding higher FI (0.94) suggests that the test item was extremely easy for all examinees who attempted this question and hence is extremely poor test item. The combined DI of "Section C" is 0.98, due to three reasons; first one is that the score of Question 1, was excluded from the overall score as it was received from a different source other than that from whom score of other questions was received, the second one is that the overall higher facility indices of question 2 "Section B" and questions (3-9) "Section C" are (0.75 & 0.59) and their discrimination indices (0.95 & 0.98) and third one was that the combined contribution of the score of "Section C" was 50 percent of the overall score used for calculations. This suggests that "Section C" has a better DI as compared to "Section B" and questions contained in this section were more challenging.

Specific Responses to Questions of "Section C"

Ouestion 3

If θ is measured in radians then show that, $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$

This question required from the candidates to carry out the geometrical construction of a right angled triangle by taking a sector of a unit circle with its centre at the point O. Let OAB represents a sector of a unit circle with centre O, central angle $\theta(acute)$

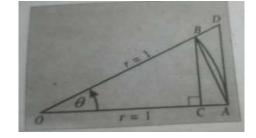
radius:
$$|OA| = |OB| = 1$$
 unit

Join A to B. Draw
$$\overline{BC} \perp \overline{OA}$$
.

In Right
$$\triangle OBC$$
, $\sin \theta = \frac{\left| \overline{BC} \right|}{\left| \overline{OB} \right|} = \left| \overline{BC} \right|$

produce \overline{OB} to D so that $\overline{AD} \perp \overline{OA}$

In Right
$$\triangle OAD$$
, $\tan \theta = \frac{|\overline{AD}|}{|\overline{OA}|} = |\overline{AD}|$



Candidates were expected to express the comparison of the areas of the triangles formed with

the area of the sector

Area of $\triangle OAB < Area$ of $\sec tor OAB < Area$ of $\triangle OAD$

$$\frac{1}{2}|OA||BC| < \frac{1}{2}|OA|^2 \theta < \frac{1}{2}|OA||AD|$$

They were required to simplify and express the inequalities in terms of trigonometric ratios by taking the reciprocals of all the fractions

 $\sin \theta < \theta < \tan \theta$

$$1 < \frac{\theta}{\sin \theta} < \frac{1}{\cos \theta}$$

$$1 > \frac{\sin \theta}{\theta} > \cos \theta$$

$$\therefore 0 < \theta < \frac{\pi}{2}$$

They were also required to take the limits as $\theta \to 0$

$$1 > \lim_{x \to 0} \frac{\sin \theta}{\theta} > \lim_{x \to 0} \cos \theta$$
$$1 > \lim_{x \to 0} \frac{\sin \theta}{\theta} > 1$$

Candidates were expected to apply the Sandwich Theorem to prove the result

By Sandwich Theorem,
$$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$$

Question asked was of mixed cognitive levels (knowledge, understanding and application) and was attempted by around 53.71 percent of the candidates. It required the candidates to recall (knowledge level) the geometrical construction to form two right angled triangles with the radii of a sector of a unit circle having angle ϑ such that one triangle is having area smaller while the other one greater than the sector. Candidates were required to express these areas in the form of inequalities and to simplify and express (understanding level) these inequalities in the form of trigonometric ratios. They were also required to take limits as $\vartheta \to 0$ and to apply (application level) Sandwich Theorem to prove the required relation. Almost all candidates who attempted this question performed extremely well except for a very few (10-15%) who faced difficulties in taking limits and applying the Sandwich Theorem (application level) to get the relation as asked in the question. Candidates on average secured 6.49 (81.09%) marks and around 58.93 percent out of them secured full (100%) marks in this question.

Question 4

Find
$$\frac{dy}{dx}$$
 if $y = x \sin^{-1}\left(\frac{x}{a}\right) + \sqrt{a^2 - x^2}$

This question asked the candidates to differentiate the given function by applying the procedure and formulae as under,

$$y = x \sin^{-1}\left(\frac{x}{a}\right) + \sqrt{a^2 - x^2}$$

$$\frac{dy}{dx} = \sin^{-1}\left(\frac{x}{a}\right) + x \frac{1}{\sqrt{1 - \frac{x^2}{a^2}}} \cdot \frac{1}{a} + \frac{1}{2\sqrt{a^2 - x^2}} \left(-2x\right)$$

They were required to simplify and rearrange the equation as under,

$$\frac{dy}{dx} = \sin^{-1}\left(\frac{x}{a}\right) + x\frac{a}{\sqrt{a^2 - x^2}} \cdot \frac{1}{a} - \frac{x}{\sqrt{a^2 - x^2}}$$

$$\frac{dy}{dx} = \sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{\sqrt{a^2 - x^2}} - \frac{x}{\sqrt{a^2 - x^2}}$$

$$\frac{dy}{dx} = \sin^{-1}\left(\frac{x}{a}\right)$$

Question asked was of knowledge-cum-understanding level and was attempted by around 48.98 percent of the candidates. It required from the candidates to recall (knowledge level) the formulae and procedure for differentiation. They were also required to simplify and rearrange (understanding level) the equation. Out of 48.98 percent of the candidates who attempted this question majority of them answered it correctly while around 30-35 percent of them gave vague, incorrect and incomplete answers. Candidates on average secured 5.13 (64.07%) marks and around 34.53 percent out of them secured full (100%) marks in this question.

Question 5

Evaluate
$$\int \sqrt{a^2 + x^2} dx$$

This question required the candidates to carry out integration by parts as under,

Let
$$I = \int \sqrt{a^2 + x^2} dx$$

 $I = x\sqrt{a^2 + x^2} - \int \frac{1}{2\sqrt{a^2 + x^2}} 2x.(x) dx$
 $I = x\sqrt{a^2 + x^2} - \int \frac{x^2}{\sqrt{a^2 + x^2}} dx$

They were required to add and subtract a^2 and simplify the equation as under,

$$I = x\sqrt{a^2 + x^2} - \int \frac{a^2 + x^2 - a^2}{\sqrt{a^2 + x^2}} dx$$

$$I = x\sqrt{a^2 + x^2} - \int \sqrt{a^2 + x^2} dx + a^2 \int \frac{1}{\sqrt{a^2 + x^2}} dx$$

They were required to replace I for its value and use the formula for integration

$$I = x\sqrt{a^2 + x^2} - I + a^2 \ln |x + \sqrt{a^2 + x^2}| + c'$$

Candidates were expected to simplify the equation and to find I as under,

$$2I = x\sqrt{a^2 + x^2} + a^2 \ln |x + \sqrt{a^2 + x^2}| + c'$$

$$I = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\ln\left|x + \sqrt{a^2 + x^2}\right| + c \quad \text{where } c = \frac{c'}{2}$$

Question asked was of mixed cognitive levels (knowledge, understanding and application) and was attempted by around 32.16 percent of the candidates. It required them to recall (knowledge level), the formulae and procedure for integration by parts and an understanding in arranging the equation by addition and subtraction of, α^2 for applying the integration formulae besides simplifying the equation to find the value of "I". Out of those candidates who attempted this question majority of them answered extremely well. However, almost 50-

55 percent out of them faced difficulty in carrying out integration by parts and in simplifying the equation to find the value of "I". Candidates on average secured 4.45 (55.57%) marks and around 13.78 percent out of them secured full (100%) marks in this question.

Question 6

Find h such that the points A(h,1), B(2,7) and C(-6,-7) are the vertices of a right triangle. In this question the candidates were required to find the lengths of all the three sides of the triangle as under,

$$A(h,1), B(2,7), C(-6,-7)$$

$$|AB| = \sqrt{(2-h)^2 + (7-1)^2}$$

$$|BC| = \sqrt{(-6-2)^2 + (-7-7)^2} = \sqrt{260}$$

$$|CA| = \sqrt{(h+6)^2 + (1+7)^2}$$

They were required to apply Pythagoras Theorem by taking $m \leq BAC = 90^{\circ}$ to find the value of h as under,

$$|BC|^{2} = |AB|^{2} + |CA|^{2}$$

$$260 = (2-h)^{2} + (7-1)^{2} + (h+6)^{2} + (1+7)^{2}$$

$$h^{2} + 4h - 60 = 0 \implies h = 6 \text{ or } -10$$

Candidates had the option to take any of the other two angles alternately as right angle and applying Pythagoras Theorem would get different values of h as under,

Alternately took m <ABC = 90° , then value of h = 12.5

Alternately took m <ACB = 90° , then value of h = -20

There were other Alternate Methods which the candidates could apply to find the value of h, two out of them are as under,

- i) Found slopes of two sides of the triangle
- ii) Applied the product rule of slopes perpendicularity

Question asked was of mixed cognitive levels (knowledge, understanding and application) which required the candidates to recall the procedure for finding length between two points and understanding to apply Pythagoras Theorem by taking any of the vertex angle as 90° and by simplifying the equation to find the value "h" as desired by the question. Around 30-35 percent of the candidates found the value of "h" by other methods and could successfully secure maximum marks. This was one of the most popular choice questions from this section which was attempted by almost 77.11 percent of the candidates who on average secured 6.60 (82.45%) marks and around 66.18 percent out of them secured full (100%) marks in this question.

Question 7

Find an equation of a parabola with focus (-1,0) and vertex (-1,2)

This question required the candidates to state that as the x-coordinates of focus F(-1,0) and vertex V(-1,2) are the same so F lies below the vertex V. Equation of required vertical

parabola (opened below) is as under,

$$(x - h)^2 = -4a (y - k)$$

They were required to argue that since the coordinates of the Vertex

V (h,k) = V (-1,2) as given; so h= -1 and k=2

Candidates were required to state that, a = |VF| and calculate its value as

=\)
$$a = \sqrt{(-1+1)^2 + (0-2)^2} = 2$$

They were expected to obtain the equation of the parabola by substituting the values of h, k, and a in the general equation of parabola as under,

$$(x+1)^2 = -4(2) (y-2)$$

$$(x+1)^2 = -8(y-2)$$

Question asked was of mixed cognitive levels (knowledge, understanding and application) which required the candidates to infer from the given coordinates of focus and vertex the shape and to give the general equation of the parabola. They were also required to find the value of "a", the length of the line segment joining the vertex and the focus points. They were expected to give the equation of the parabola by substituting the values of "h, k and a" in the general equation of the parabola to get the required equation. This was the least choice questions attempted by only 6.29 percent of the candidates who on average secured 5.80 (72.54%) marks and around 52.46 percent out of them secured full (100%) marks in this question.

Question 8

Find the centre, foci, eccentricity, vertices and equations of directrices of hyperbola, $\frac{y^{2}}{16} - \frac{x^{2}}{9} = 1$

This question required the candidates to compare the given hyperbola equation with the standard equation to get the values of a and b as under,

$$\frac{y^2}{16} - \frac{x^2}{9} = 1$$
 Compared it with $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

a = 4 and b = 3

The candidates were required to give the coordinates of the centre of the hyperbola, Centre: (0,0)

They were required to calculate the value of c by substituting the values of a, and b, in equation:

 $c^2 = a^2 + b^2$ which comes out as c=5. They were expected to give the coordinates of foci by substituting value of c as: Foci: $(0, \pm c) = (0, \pm 5)$

The candidates were asked to give the relationships of eccentricity, coordinates of vertices and to calculate their values as under;

eccentricity:
$$e = \frac{c}{a} = \frac{5}{4}$$

Vertices: $(0,\pm a) = (0,\pm 4)$

They were also required to give the equations of directrices as under,

Equations of Directrices: $x = \pm \frac{c}{a} = \pm 4$

Question asked was of **knowledge-cum-understanding level** which required the candidates to infer from the given equation of the hyperbola the coordinates of the centre and by

calculating the value of "c" were also required to give the coordinates of foci, eccentricity, vertices and equations of directrices by recalling their relations in terms of "a, b and c" and to calculate them by substituting their values. This was one of the least choice questions attempted by only 9.69 percent of the candidates who on average secured 4.78 (59.71%) marks and around 27.66 percent out of them secured full (100%) marks in this question.

Question 9

Prove that the midpoint of hypotenuse of a right triangle is equidistant from its vertices

This question required the candidates to give the construction of right angled triangle on the coordinate axes by taking the vertices at the points; origin O(0,0), on the x-axis A(a,0) and on the y-axis B(0,b) and angle $M < AOB = 90^{\circ}$. They were also expected to take $P\left(\frac{\alpha}{2}, \frac{b}{2}\right)$ as the midpoint on the hypotenuse AB. By taking vertex O(0,0) as reference point they were required to determine the position vectors (p.v.).

p.v. of A =
$$\overrightarrow{OA} = \begin{bmatrix} a, 0 \end{bmatrix}$$
 p.v. of B = $\overrightarrow{OB} = \begin{bmatrix} o, b \end{bmatrix}$
p.v. of P = $\overrightarrow{OP} = \begin{bmatrix} \frac{a}{2}, \frac{b}{2} \end{bmatrix}$

Candidates were expected to determine distances of AP, BP and OP as under,

$$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = \left[\frac{a}{2}, \frac{b}{2}\right] - [a, 0] = \left[-\frac{a}{2}, \frac{b}{2}\right]$$

$$\left|\overrightarrow{AP}\right| = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \frac{1}{2}\sqrt{a^2 + b^2}$$

$$\overrightarrow{BP} = \overrightarrow{OP} - \overrightarrow{OB} = \left[\frac{a}{2}, \frac{b}{2}\right] - [0, b] = \left[\frac{a}{2}, -\frac{b}{2}\right]$$

$$\left| \overrightarrow{BP} \right| = \sqrt{\frac{\alpha^2}{4} + \frac{b^2}{4}} = \frac{1}{2} \sqrt{\alpha^2 + b^2}$$

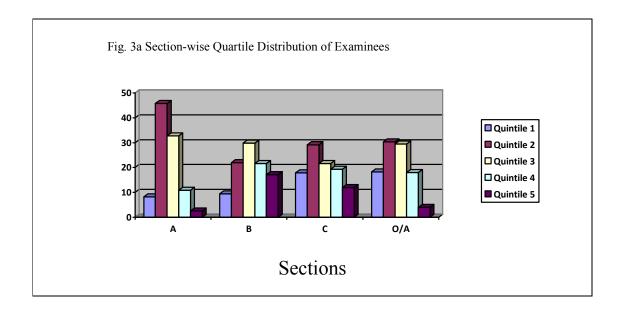
$$|\overrightarrow{OP}| = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \frac{1}{2}\sqrt{a^2 + b^2}$$

Candidates were required to argue that as $|\overrightarrow{AP}| = |\overrightarrow{BP}| = |\overrightarrow{OP}| = \frac{1}{2} \sqrt{a^2 + b^2}$ so the midpoint of hypotenuse of a right triangle is equidistant from its vertices.

Question asked was of mixed cognitive levels (knowledge, understanding and application) which required the candidates to recall the construction of a right angled triangle on a coordinate system and to give the position vectors of the other two vertices besides midpoint on the hypotenuse by taking one of the vertices at the origin. By calculating the distances of the point "P" from the vertices they were required to prove that the "P" is equidistant. This was one of the most popular choice questions attempted by around 47.22 percent of the candidates and majority of them answered it successfully. They on the average secured 6.08 (75.96%) marks and around 61.79 percent out of them secured full (100%) marks in this question.

Summary

The higher facility and discrimination indices suggest that the paper was reasonably good and students performed quite well in answering the questions. However, since all questions were given from the prescribed book so the students who had spent more time in preparation performed better in all the questions. The positive value of the discrimination index in all questions reflects that those students who were overall in the upper bracket performed better than those who were in the lower bracket in all questions. Further, as all the questions were given from the prescribed book, which are covered in classroom teaching as a routine so the students had to mere recall them (knowledge level). The question-wise examinees falling in different quintiles are given in the figure (3a,3b) and percentage of those securing 100 percent marks in each question are given in figure 4 below:



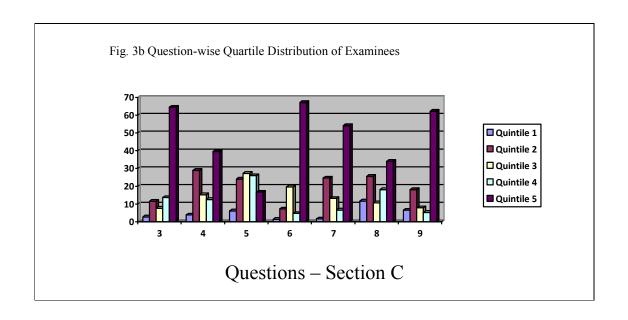


Figure 4 Question-wise percentage of candidates securing full (100%) marks.

